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Gödel's Theorem in Perspective

Interest continues in a mathematical theorem first stated by Kurt Gödel in 1931. This paper reviews a recent popularization of the theorem by Raymond Smullyan in his paperback 'Forever Undecided'. Considerable use is also made of Douglas Hofstadter's account in 'Gödel-Escher-Bach'. The key ideas of consistency, formal systems, self-reference, provability and truth are developed, and an outline is given of Gödel's two theorems and those of Henkin and Löb. Applications and analogies are then distinguished and discussed, including computer theory, the human mind, cosmology, the work of Rosen in biology, and the status of the Church–Turing thesis. Finally, attention is paid to some implications for Christian belief: the nature of man, is belief self-fulfilling, is the Bible self-referent, and the importance of the personal encounter.

Key Words: Artificial Intelligence, Church–Turing Thesis, decidability, formal systems, Gödel, Hofstadter, image of God, proof, Smullyan, truth.

1 Introduction

It is now nearly sixty years since Kurt Gödel first published his famous paper 'On formally undecidable propositions'[1] in which he proved that arithmetic, and other larger mathematical systems, cannot prove their own consistency, and must contain propositions that are undecidable—i.e., which cannot be either proved or disproved by arithmetical means. Ever since the unknown Pythagorean discovered that no unit of measurement, however small, could measure simultaneously the side and diagonal of a square, incompleteness theorems of this kind have caused a stir in the mathematical pond. But the ripples set up by Gödel's boulder have spread far beyond mathematics, into computer science, philosophy, psychology, theology and other areas. There have been many popular accounts of these developments; the most outstanding and the widest-ranging is Douglas Hofstadter's *Gödel–Escher–Bach*,[2] a monumental work, popular only in the sense that specialized mathematical knowledge is not needed, but which no serious student can pass by. Recently, however, another mathematical logician, Raymond Smullyan, has written what he calls 'a puzzle guide to Gödel',[3] which is rather more accessible, and this I shall consider in some detail, though occasionally I shall need to refer to Hofstadter.

1 Gödel, Kurt On formally undecidable propositions (1931) (English Translation, Basic Books, N.Y. 1962).

2 Hofstadter, Douglas *Gödel–Escher–Bach: an Eternal Golden Braid* (Penguin, 1980) hereafter 'H'.

3 Smullyan, Raymond *Forever Undecided—a puzzle guide to Gödel* (Oxford, 1988) hereafter 'S'.

2 Smullyan's Archipelago

'*Forever Undecided*' is the latest of a number of paperbacks by this author which reveal his remarkable gift for putting difficult ideas in popular and easily assimilable form. What is also clear is his caution and restraint in handling the analogy between results in mathematical logic which he discusses and matters in psychology and theology which he uses as illustrations. Many popularizers have extrapolated theorems in formal logic into pronouncements about God and the human mind, about cosmology and quantum theory, which I would consider unjustified; there are connections, but *theology and nature are not formal systems*, and the connections are not simple ones.

Smullyan's method is to use logical puzzles to stimulate the reader and to lead him by easy stages into the world of mathematical logic. In his books he introduces us to an archipelago of islands, inhabited by knights (who always tell the truth), knaves (who always lie), normals (who may do either), zombies (who refuse to speak English) and other devious types. In *Forever Undecided* we spend most of our time on a single island inhabited only by knights and knaves, but visited by human reasoners of various degrees of sophistication, who correspond to different formal systems. By a chain of problems, to nearly all of which solutions are given, he unravels the complexity of the theorems of Gödel, Löb, Rosser, Kripke and others, culminating in an examination of the meaning and application of modern modal logic. The book can be strongly recommended as a non-technical introduction to this area; it includes a fairly formal treatment of propositional calculus, an outline of axiom systems, a simple example of arithmetization, and an exploration of self-reference, provability, decidability and consistency which are vital elements of the theory.

We consider in turn some of these elements, which are key ideas in the approach to Gödel's theorems.

3 Key Ideas in Gödel's Theory

(a) Consistency

We say that a logical system is inconsistent if by valid logical processes one can deduce in it a proposition *and* a denial of that proposition—p and not-p. It doesn't matter what p is, because, if we can deduce p and not-p, we can then also deduce q and not-q for any other proposition q. Logicians have a name for this contradictory combination (p and not-p); it is 'eet', the opposite of 'T' for truth. A system is consistent if eet cannot be proved in it.

Now it so happens that a classical paradox which is basic to Gödel's argument occurs in the Bible. A Cretan poet called Epimenides wrote the line 'Cretans are always liars', which is quoted in Titus 1:12 as a warning for Titus to treat his congregation with caution. Now, if this is true, Epimenides himself is a liar who for once is telling the truth, which—note the 'always'—is a logical contradiction: it is eet. So, if Titus is a consistent reasoner and unwilling to accept eet, he must conclude that Epimenides is lying and there is at least one truthful Cretan. This is fortunate for Titus, since he can appoint at least one bishop, but unfortunate for the author of the letter, since he accepts eet (unwittingly, of course!) in the next verse by writing 'this saying is true'. So the author has become inconsistent.

It is important to realize both that acceptance of eet is one of the usual definitions of inconsistency, and also that those who accept eet can prove anything, including their own consistency! There is an anecdote in one of Smullyan's earlier books[4] which illustrates this; Smullyan attributes it there to Bertrand Russell, but I have always heard it told of Prof. G. H. Hardy at Trinity College, Cambridge. After dinner there one night he was challenged by a fellow don: 'They tell me, Hardy, that if you accept a false proposition you can prove anything. Is that true?' 'Yes', said Hardy. 'Very well then, if I tell you that $2 + 2 = 5$, can you prove that McTaggart over there is the Pope?' Quick as a flash came the answer: '$5 = 2 + 2$, therefore $5 = 4$. Subtract 3 from each side, therefore $2 = 1$. McTaggart and the Pope are two, therefore McTaggart and the Pope are one'.

To return to Smullyan's island of knights and knaves; no one on this island can say 'I am a knave'; a knight cannot, for it would be a lie, and a knave cannot, for it would be true. But someone might say 'all of us are knaves' (the Epimenides paradox), which would prove that he was a knave and there was at least one knight on the island. Now take this a step further. Suppose someone says to us 'If I am a knight, there is gold on this island'. What do we conclude?[5] Well, if he is a knave, this has to be a lie, and the only way this can be so is for him to be a knight *and* for there to be no gold on the island. But this is eet, so he *is* a knight, and there is gold on the island. (We note in passing that, if he had said 'I am a knight *if and only if* there is gold on this island', we could have concluded only that there is gold on the island. Every knave can say 'I am a knight.'!)

Of course, the business about the gold is not significant here; if p is any proposition whatever and an islander says 'If I am a knight, then p', then we are bound to believe that p is true. This is a very unrealistic situation and it arises from the rules of the island: everyone either always speaks the truth or always lies. Smullyan is using the island as a model of a system of statements which are either true or false; the visitors are operating the logical rules of the system, and what they conclude to be true are the provable statements *in the system*. It will help to define some things more clearly.

(b) A Formal System

A formal system S is a system based on a finite set of symbols (usually quite a small set) which are combined together to form strings or words. Certain strings are given as starters, or *axioms*, and the system is built up from them

4 Smullyan, R. 'What is the name of this book?' (sic) (Penguin, 1981) p. 219.
5 S, pp. 17–20.

by given rules. Symbols, axioms and rules are of course usually chosen to have *meaning* in a definite context, e.g., in logic, where ~ will mean 'not', or in arithmetic, where 0 will mean zero and s 'successor', so that s0 means 1, and (a)~sa = 0 is an axiom, where (a) means 'for all a'. Without axioms there can be no system—one cannot even get started. Arithmetic, algebras, geometries, models of physical theories, all begin with axioms—unproved assumptions—from which the model is built up. This obvious truth is sometimes stated as a consequence of Gödel's Theorem, but that is a far more subtle matter.

(c) Another Essential Idea in Gödel's Argument is that of Self-Reference

We have already met partial self-reference in Epimenides' statement above. Epimenides is a Cretan referring to Cretans. But the sentence does not actually refer to itself, except by implication. No finite sentence or formula can refer to itself explicitly—i.e., make a statement about itself which contains a statement of itself—for then it would be longer than itself, which is impossible (unless it is infinite!). But it can refer to a description of itself: THIS SENTENCE IS TRUE is a crude example, which requires us to know what we mean by 'this sentence', but of course conveys no information whatsoever. THIS SENTENCE IS FALSE is a self-contradictory example. A better example which does not require us to know what 'this sentence' is has been given by Hofstadter;[6] it is:

"YIELDS FALSEHOOD WHEN PRECEDED BY ITS QUOTATION" YIELDS FALSEHOOD WHEN PRECEDED BY ITS QUOTATION.

Hofstadter calls this process '"to quine a phrase", to quine a phrase'. (There is a logician named W. van O. Quine.) The essential idea here is that a phrase is encapsulated by being put in quotes, and is then made the subject of a sentence which is itself, now given meaning. The phrase enters the full sentence in two ways: as a noun-phrase whose meaning is irrelevant, and as a meaningful predicate. What Gödel wanted to do was to find a formula expressible in a formal system, whose 'quinification' was statable in the system, and whose meaning, in a standard interpretation of that system, would be '"This is not provable" is not provable!'. He took S to be arithmetic, formalized from axioms, and devised a method of numbering the statements of S in such a way that the complete statement could be recovered simply by factorization of its number; such factorization could then be interpreted as a statement of number theory. By giving statements *Gödel numbers* in this way they can be encapsulated in arithmetic and so referred to; in fact, not only individual statements, but whole chains of statements constituting, say, a *proof* can be numbered in this way.

6 H, pp. 435–7.

(d) Belief and Truth

Before I go on to a more formal statement of Gödel's theorems, I would like to return briefly to Smullyan. He invites us to consider the statement made by an islander: 'If I am a knight, you will not believe that I am'. Now, as we argued in (a), if we believe he is a knight, then what he says is true and we don't believe that he is! So, if we are *normal*, and believe what we believe, we both believe that we believe he is a knight, and we believe that we don't believe this, so we are inconsistent. If we are to be consistent, we have to believe he is a knave, but is speaking the truth. So there are true statements which don't conform to the rules of the island. If we drop the analogy and replace 'believe to be true', by 'provable', and 'normal' by 'accepting what is provable as true' we are able to see that 'you will never believe that I am a knight' is a Gödelian statement in disguise. When, in John 8:45, Jesus is reported as saying 'because I tell you the truth, you do not believe me', he is not posing a logical conundrum, but he *is* challenging the consistency of his hearers.

4 Gödel's Theorems

These can now be more formally stated. The two crucial steps were the codifying in Gödel numbers of the statements:

(Proof)—'a is the Gödel-number of a proof whose last line has Gödel-number b'

(Quinification)—'c is the Gödel-number of the formula obtained from the formula with Gödel-number d, containing a variable x, when x is replaced by the numeral for d'.

He then found a statement G, *with Gödel-number* g, whose interpretation, when decoded, says:

(G) There is no proof in S of the formula with Gödel-number g.[7] (S here is the formal system with which we started—formalized arithmetic, but it can in principle be any formal system which is equally powerful.) It is important to realize that G is actually a statement in the symbols of S, which can be decoded to give a statement about ordinary whole numbers. So G is saying 'I am a statement about numbers which cannot be proved using the logical processes of arithmetic'.

Now, if G were provable in S, then we would have proved the theorem that G is *not* provable, which is eet. It follows, if S is consistent, that G is not provable, which means—since this is what the interpretation of G says—that G is true. But we cannot prove G disprovable either; i.e., we cannot prove ~ G. For ~ G means 'G is provable' and this is false, and if S is consistent, we cannot prove any false statements in S. So G is neither provable nor disprovable, but G is true. And G is a (very complicated) logical statement about numbers, in coded form. So we have:

7 H, pp. 447f.

Gödel's First Theorem

Every formal axiomatic system S, large enough to contain arithmetic, contains statements which are not provable in S, and, if S is consistent, they are not decidable in S.[8]

Now we have just proved that, if arithmetic is consistent, G is true. So, if it were possible *to prove in arithmetic* that arithmetic was consistent, it would be possible to follow this proof by the above argument and so to prove that G is true. But G says precisely that such a proof is impossible. Therefore we have

Gödel's Second Theorem

If such a system S is consistent, then it is impossible to prove it consistent by methods within the system S.[9]

Of course, if S is inconsistent, i.e., if S includes eet, then *anything* can be proved in S, including the consistency of S! (Remember McTaggart!) Are we in danger of this happening when we do our arithmetic? No, for, mercifully and not surprisingly, *we* can prove arithmetic consistent, but the proof will involve ideas which are not formally expressible in arithmetic, such as transfinite induction.[10] On reflection, this is fairly to be expected, but the consequences are important and far-reaching. We might be tempted to think that Gödel is talking about something very abstract and comfortably remote from straightforward mathematical research; but we would be wrong. For it can be shown that, corresponding to Gödel's undecidable sentence G, there is a perfectly ordinary, though exceedingly complicated, algebraic equation E. If we could find a solution to E in whole numbers then from that solution we could calculate the Gödel number of a proof that E had no solutions.[11] So, while the interpretation of E on the decoded level is true, no computer program written to find a solution to E will ever come to a stop. Hofstadter likens this vividly to a record, which, when played, causes vibrations which break the record-player it is played on.[12]

We need to understand clearly that while Gödel's formula is undecidable in S, the fact that it is *true* comes from our understanding of its interpretation, which is not in S, but in our reasoning *about* S, i.e., in what is usually called *metamathematics*, and the proof of Gödel's theorem is metamathematical also. (Some mathematicians believe that all undecidable problems in mathematics can be interpreted as Gödel sentences in some code—but this seems to me to be 'hyper-undecidable'!)

We note too here for reference later on that the logician Henkin showed how to construct a 'self-reinforcing' sentence H in a system S, unlike the

8 S, pp. 173–8, H, pp. 271–2, Chapter XIV.
9 S, pp. 102, 253.
10 S, pp. 110–11, Mendelson, *Introduction to Mathematical Logic*, pp. 148, 248–271 (van Nostrand, 1964).
11 H, p. 460.
12 H, p. 271.

'self-destructive' sentence G. If H has Gödel-number h, then H, when decoded, says:

(H) There is a proof of S in the formula with Gödel-number h.[13] At first sight, we might think that H could easily be false and unprovable, but Löb succeeded in proving that, under a simple condition on S (that S is *reflexive*) which is true for arithmetic, H is provable in S and true. A proof, using Smullyan's reasoning analogy, will be found in his book.[14] A reflexive system corresponds to an island in which, whatever proposition p you make, someone can always be found to say to you 'If you ever believe I am a knight, then p is true'. Most of us would I think conclude that this must be an infinite island full of strange people.

5 Applications

Gödel's Theorem is frequently referred to in the literature in connection with topics far removed from the field of mathematical logic. It seems to me that we need to distinguish between applications and analogies. Application means that we have a mathematical model of a body of material phenomena resting upon axioms which correspond to experimentally verifiable facts—e.g., Newton's laws—whose logical consequences enable us to make predictions. The model will be useful as long as these predictions agree with experimental results, i.e., there is *isomorphism* between model and observed data. 'Meaning' for us is also an isomorphism—in this case with our own conscious experience, as Hofstadter himself has pointed out.[15] But there are many instances where we might like to use the ideas and concepts which Gödel had brought to us, but there is as yet no formal isomorphic model which is well-attested by experiment. In fact this has been done where it is at least questionable whether a formal model of this kind *can* be set up, since concepts like 'to every number there is a successor' and the existential and universal quantifiers will have to be included. These instances must be called *analogies*, rather than applications. True applications include three which are of interest. One is an extension to logic known as modal logic, especially to that form of it which distinguishes truth from necessary truth, which is the subject of Smullyan's final chapters. But whether or not all that is true is necessarily true seems still to be undecidable. The second is to set theory and the ramifications of transfinite numbers. According to Devlin,[16] the work of Paul Cohen has led to a method of showing that many classical problems in this branch of mathematics are in fact undecidable. The third is to computer theory. A computer is certainly geared to run on strict logical rules, and an important consequence of Gödel's Theorem is Turing's Theorem: There is no way of distinguishing between computer

13 S, p. 142; H, p. 541.
14 S, p. 146.
15 H, p. 50.
16 Devlin, Keith, *Mathematics: the New Golden Age* (Penguin, 1988), pp. 46–7.

programs which will inevitably stop, whatever their input, and those for which there is at least one input for which they will never stop. Of course many programs can easily be put in one class or the other, but there is no procedure which will infallibly decide this. Put another way, this means that there is no possibility of devising a computer program which will infallibly distinguish theorems from non-theorems or true statements from false ones.[17]

There are many nowadays, especially those working in Artificial Intelligence, of whom Hofstadter is one, who would include in the list of applications the reasoning powers of the human mind. The last 200 pages of Hofstadter's book are devoted to this; we must be content with a brief consideration. We can agree that at the lowest level, the neural activities of the brain are similar to the mechanistic electronic behaviour of computer hardware. The question is, how far up does this similarity extend? This is the subject of the Church–Turing Thesis which was put forward independently by two mathematicians in the 1930s—Alan Turing and Alonzo Church. Hofstadter states this in its strongest form as follows:[18]

> Mental processes of any sort can be simulated by a computer program whose underlying language is of power sufficient to program all partial recursive functions. (A partial recursive function is one which is computable by a program which can, but need not necessarily, terminate.)

If we accept this, it means that our thinking, hoping, imagining are like the software which controls and is also to some extent controlled by the underlying neural chemistry; our memories are a static or dynamic databank filled by our experiences and programmed by our education. But what about our self-awareness and our sense of freedom of choice? Hofstadter and others would appear to argue[19] that these arise automatically at some level of complexity, when a high level in the hierarchy of control becomes interlinked with a much lower level—an enormous extension of the idea of a statement including its own Gödel-number. This would seem to imply that at some stage of development a computer will be able to know itself, organize itself and take action, subject of course to its dependence on power-supply, space and time. This is an alarming prospect—is it true?

At least one attempt has been made to refute this, using Gödel's Theorem itself, by J. R. Lucas[20] in 1961. Roughly speaking, his argument is that if the mind is isomorphic to a formal system it must contain Gödelian situations which are not decidable in the system—i.e., in the mind. But this is false, because our experience is that we (and by 'we' we mean a sufficiently ingenious mind) can always 'out-Gödel Gödel'; we can invent a larger system by adding an axiom or stepping up a level, just as

17 H, pp. 579ff.
18 H, p. 578.
19 H, pp. 708ff.
20 Lucas, J. R., *Minds, Machines and Gödel*, *Philosophy* 36 (1961) p. 112.

we can show that arithmetic is consistent by moving up one from arithmetic. Hofstadter rejects this argument[21] by pointing out that there is an implicit 'and so on' here, leading to an infinite regress unattainable by a finite mind. With this rejection I agree; Lucas has slipped in what he wants to prove—the infinite capabilities of the human mind. I find myself in good company—for example, Paul Davies[22] has a good discussion of this argument and its Christian implications in 'God and the New Physics'. Polkinghorne, however, seems to accept it when he says; quoting Rucker:[23] 'No finitely programmed machine can ever exhaust the richness of the mental and physical world we inhabit'. Certainly Hofstadter is *practically* right: no single mind can embrace all Gödelian extensions. Mendelson finds it necessary to remind us of this when he says:[24] 'Some of the computations needed to obtain the values of a partial recursive function involve so many steps that the human race may not exist long enough to carry them out'. I must say that when one thinks about these things for very long one becomes increasingly conscious of human finitude. The fact that Gödel, Escher, Turing themselves ended up mentally disturbed is itself disturbing.

6 Analogies and Possibly False Applications

We may appropriately begin this section with a quotation from Hofstadter.[25]

> 'Gödel's Theorem shows that there are fundamental limitations to consistent formal systems with self-images. But is it more general? Is there a "Gödel's Theorem of psychology", for instance?
>
> If one uses Godel's Theorem as a metaphor, as a source of inspiration, rather than trying to translate it literally into the language of psychology or of any other discipline, then perhaps it can suggest new truths in psychology or other areas. But it is quite unjustifiable to translate it directly into a statement of another discipline and take that as equally valid. It would be a large mistake to think that what has been worked out with the utmost delicacy in mathematical logic should hold without modifiction in a completely different area.'

This is a timely warning which is frequently ignored. Smullyan does not do so, but his analogy is interesting. He replaces 'provable in a system' by 'believed by a reasoner'. Gödel's string becomes 'You will never believe that I am a knight', as we have seen. He tells us in the preface that he has used this method largely for psychological reasons—it 'grabs' his students more effectively, and it certainly makes entertaining reading. It also opens the way to exercises (which are frequent) about a student and his theology

21 H, pp. 471ff.
22 Davies, Paul, *God and the New Physics* (Dent, 1983) pp. 941ff.
23 Polkinghorne, J., *Science and Creation* (SPCK, 1988) p. 82.
24 Mendelson, op. cit. p. 227.
25 H, p. 696.

professor, and the idea that 'if you believe you will be saved, then you will be saved' is left dangling in the background as a possible example of a self-fulfilling belief. In the foreground is the less controversial statement made by a doctor to his patient: 'If you believe the cure will work, then it will work'. Belief in such statements would then, in the analogy, lead to important results—we are in the world of Henkin and Löb—but to transfer them to real life involves acceptance of the Church-Turing thesis, and acceptance of the rules of the island about people in general, to say nothing of the reduction of Christian belief to logical assent and salvation to peace of mind.

Another application which is frequently attempted is to cosmology. A relevant example which is somewhat astonishing comes from a distinguished theologian with a reputation for logical clarity in scientific matters.[26] In the course of an argument about the contingent nature of the universe, with which I wholly agree, we find this sentence:

> 'We must recognize ... that if our mathematical propositions are certain, they are not true, and that if they are true, they are not certain, and that the universe far from being infinite is finite, though unbounded or open, as also becomes clear when Gödel's incompleteness theorem is applied to the universe as a whole.'

Frankly, I can make no sense of this at all. I do not understand the distinction between truth and certainty; if 'certain' means 'provable', then I agree that truth is wider than provability, but they are certainly not antithetic as stated here. Mathematics must take all that is provable as true, and only some propositions are true but not provable. Gödel has not reduced all mathematics to total absurdity! Again, 'open' is only equivalent to 'unbounded' in a technical topological sense, and is rather misleading in this context. Finally, I cannot see that Gödel's Theorem tells us anything at all about the geometry of the universe—e.g., that it is topologically equivalent to the surface of a four-dimensional sphere. If a formal model could be made of the whole universe (which Torrance and I would both want to deny), then Gödel would tell us that questions could arise *in the model* which are undecidable. But even if the universe were infinite—and I agree that it is probably not—there is no reason why these questions should correspond to anything in the material universe. While everything in the universe would have to be accessible to a faithful model, not everything in the model need have a material counterpart.

Of course, if we accept the argument of Lucas, that Gödel's Theorem compels us to agree that the mind is not reducible to a formal system—i.e., to reject the Church–Turing thesis—then the universe itself, containing human minds, cannot be modelled by a formal system—specifically, by Newtonian laws of causation. This, I suspect, is Torrance's real, though unexpressed, argument. It is certainly the position of Jaki, who wishes to

26 Torrance, T. F., *The Christian Frame of Mind* pp. 36–7.

deny the claim that there can ever be a unified theory of the universe which is *necessarily* true—that this is the only possible way of building a universe.[27] Here I think Gödel's second theorem can legitimately be applied. Jaki's argument goes like this: such a theory can only be proved consistent by stepping outside the theory—but if it is the only true theory of the whole universe there is nowhere else to step. So it cannot prove its own consistency, and an event might occur in it which would overthrow it and a new theory would have to be developed—i.e., it is not *necessarily* true. Furthermore, any experiment intended to establish it would have to be of eternal duration. The argument in this form seems to be valid, and it is independent of Lucas and the Church–Turing thesis. Of course it does not answer the question whether there is only one sort of universe that can possibly give rise to intelligent minds to observe it (the weak anthropic principle) but it does seem to make the strong form of that principle even harder to believe.

Before we leave this it is as well to recall that there is considerable scientific opposition, notably among biologists, to the Church–Turing thesis itself. Rosen and his school[28] have argued that living things are characterized by what we may call 'intention'; their activities are partly directed by the fact that they will produce a desired result, in fact by 'telic' considerations. We need, he argues, to go back to Aristotle's concept of *final* causes which Newton expelled from his strict causal system of the physical world. Models which admit this concept he calls *complex* systems; a typical case is the cell in which the genetic code system directs the activities of the structure which include the replication of the genetic information itself. Such models cannot be simulated in a mathematical machine, and this is established by an argument which uses the analogy of Gödel's theorem itself.[29] I hope that a mathematical biologist can tell us more about this theory, but if it is acceptable then the Church–Turing thesis has gone out of the window. What seems to have come in is a form of vitalism, or at least the idea of a programmed response to a possibly unprogrammable environment which may on the human level involve freedom of choice. This is a convenient place to stop—this *is* a halting program!—and conclude with some consideration of the relevance of what we have been saying to the Christian faith.

7 Gödel and the Christian Faith

(a) The Noetic World

The term comes from Polkinghorne[30] by extrapolation from Gödel's distinction between truth and provability. Here the mind can argue outside a system in which some things are logically not provable. We can join with

27 Jaki, S. L., Review of Hawking, *A Brief History of Time*, in *Reflections*, Vol. 7 No. 2, 1988.
28 Rosen, R., *Theoretical Biology and Complexity* (Orland, 1985).
29 Rosen, R., *Newton to Aristotle*, Casti and Karlqvist ed. (Birkhäuser, 1989) pp. 32ff.
30 Polkinghorne op. cit., Chapter 5.

him in agreeing that there are many such things which enter our mental experience. In any infinite formal model it is possible to conceive of a situation, consistent with the rules of the model, such that no finite set of experiments could ever decide whether it could arise or not. Such concepts belong to the noetic world, along with the more everyday objects of our mental experiences and imaginations, which is thus not limited to what can be shown on a computer. Polkinghorne argues for the reality of this world, which is not limited to the world of material experiences, and thus leaves open the way to admit the reality of a spiritual world, which is not unconnected with the world of matter. That we can make rational theories about the universe implies that in some way it corresponds to the rationality of our minds—we do not impose our rationality on it. It makes sense, therefore, to base both rationalities in their origin in the rationality of a creating God, who became man in Jesus and who makes himself known in the breaking of bread. Polkinghorne finds one of the criteria for the attribution of reality to the noetic world to be personal encounter. His concluding words are worth quoting:[31] 'The scientist will find in theology a unifying principle more fundamental than the grandest unified field theory. The theologian will encounter in science's account of the pattern and structure of the physical world a reality which calls forth his admiration and wonder. Together they can say with the psalmist: O Lord, how manifold are thy works! In wisdom thou hast made them all.'

This leads at once to our next consideration:

(b) The Image of God

The book of Genesis declares that God made Man in his image, and uses a world (*n^{e}shamah*)[32] for the creative breath of God which is exclusive to God and Man. What is this *n^{e}shamah* which distinguishes me from the animals? A Jewish rabbi[33] tells me that it is what enables me to talk, communicate with God and other men and women; this is certainly a large part of it, but is it all? Imagination, creativity, craftsmanship . . ., these must certainly be there, and other noetic realities. Rationality is important too, especially reasoning with a God who can say[34] 'come now and let us *reason* together'; which strongly suggests that God can provide input directly to our minds. Paul[35] asserts that we have a supreme exemplar of that image in Jesus, the second Adam, and that one concern of the divine input is the reproduction of that image in the Christian. But this seems, at least in this life, to fall short of complete self-awareness. Paul Davies[36] says Gödel's Theorem 'has been taken to imply that one can never, even in principle, understand one's own mind completely', and with this Paul the apostle

31 Polkinghorne op. cit. p. 98.
32 Genesis 1:27, 2:7.
33 Morris, Paul, Personal communication.
34 Isaiah 1:18.
35 Colossians 1:15, 3:10.
36 Davies, loc. cit.

would agree:[37] even when the Spirit of God—the divine input—works in us faith, hope and love, total self-knowledge eludes us. As reflection in a mirror is an isomorphism which falls short of identity, so I can never step outside myself and know myself completely. Self-reference is never total and does not bring total self-knowledge, but there are links with the Mainframe which will survive the destruction of the terminal. This, I think, is what Polkinghorne is saying when he describes the resurrection of the body as 'the hope of a destiny beyond death in which the information-bearing pattern . . . will be recreated in an unimaginable new environment of God's choosing'.[38]

This openness to God, the interface between the Noetic and the Divine, is what the New Testament calls faith, and this must be our next consideration.

(c) Faith and Belief

Smullyan has an interesting discussion on believing and knowing;[39] for the purposes of his analogy belief is the acceptance of something as true in the mental logical system—either an axiom (and axioms include the rules of the island) or a statement of some kind. To believe that you only believe things that are true is to be *conceited*, and to hear the statement (from an islander) 'you will never believe I am a knight' is then to condemn you to inconsistency. (This is Gödel's theorem—if we insist that all statements are provably true or false, i.e., we believe the rules of the island, we shall be inconsistent). Even in real life, pretensions to infallibility of this kind will lead ultimately to inconsistency.

But Christian belief and assurance is not like this—at least, not wholly so, for it is not unreasonable. More especially in John's gospel we find Jesus involved in logical arguments with his contemporaries. A statement such as 'If you believe that I am the Son of God (and therefore truthful) you will be saved'[40] is not part of an argument used in the proof of Löb's theorem,[41] compelling such belief, even though it looks like it. If Christian faith were a matter of correct logic, it would condemn poor reasoners to great uncertainty! One must admit that some forms of evangelistic preaching might give the impression that salvation is equivalent to acceptance of a logical system, but this is not the apostolic gospel. Logical acumen is neither necessary, nor sufficient for Christian assurance—the devils also believe and tremble. Believing in Jesus as Son of God, in Johannine terminology, includes, but is more than, believing *that* Jesus is the Son of God, as John makes clear in his letter. True faith (pistis) is not just being persuaded of facts (peithesthai), though that may be its etymological root; it involves encounter with a living Person, and increasing knowledge of him

37 1 Corinthians 13:12.
38 Polkinghorne op. cit. p. 65.
39 S, chapters 9–10.
40 John 5:24.
41 S, pp. 124–5.

which is always expressed in personal terms, and a radical redirection of life. The epigrammatic sayings in John's gospel must always be interpreted in their context and in the light of their expansion in his epistle. (This may, incidentally, be the reason why John never uses the word 'faith' in his gospel, but only the active verb 'believe'.)

For the same reason Christian faith is not just a self-fulfilling belief, a sort of Coué reinforcement of 'I am getting better every day'. (Some so-called 'healing ministries' may make it look like this!) We do not live on Löb's island. Christians find themselves in contact with a Person who radically changes their set ways of thinking, often in dramatic ways. Yet there is a measure of self-fulfilment in Christian faith—the more I am open to the God who is love, the more I find myself loving. Polkinghorne quotes in this connection a telling phrase from *The Cloud of Unknowing*:[42] 'By love he can be caught and held, but by thinking never'.

(d) The Bible

It should be clear by now that the lines of thought we have been pursuing will affect our attitude to the Bible. An extreme fundamentalist may treat the Bible as a collection of universal propositions, a formal system. To do this is to court disaster, because logical inconsistencies, which are unavoidable in a collection of historical literature, will then be treated as eet, and faith in the system is destroyed. Fear of this will then lead to untenable demands for infallibility. We have seen that it is unwise to claim infallibility in a world where people can lie; even Jesus himself was aware of this when he said 'If I bear witness to myself, my testimony is not true' (John 5:31ff.). Ultimately, our belief in him does not rest on verbal claims to infallibility, but on the reliability of his witnesses, and the direct inward testimony of God through the Holy Spirit; in short, on a personal encounter.

After all we have said, an obvious question to ask is: is the Bible self-referential? Does it authenticate itself? Some would answer immediately 'yes, it does, by using a description of itself; it speaks of itself as "the word of God"'. Of course, as James Barr pointed out some time ago[43] there is a logical difficulty here—a writer contributing to a document being developed over a period of history, cannot logically make statements which embrace the whole document, including the parts unwritten or unknown to him; we have met this problem before. But there are also theological problems: the word of God came to widely diverse people at widely diverse times and in many different ways (Heb. 1:1); how then does it come to have universal, and, in particular, present day significance? How does, e.g., Leviticus come to be 'word of God' to us? These are big questions and this is not the place to discuss them; a full treatment can be found in Dunn[44] and a simple but

42 Polkinghorne op. cit. p. 82.
43 Barr, James, *Fundamentalism* p. 78 (SCM, 1977).
44 Dunn, J. D. G., *Churchman* 1982, 96 pp. 201–225.

helpful discussion in Stein and Taylor.[45] For me, a profitable line of thought is opened by the concept of meaning which we have met with already—meaning is essentially a recognized isomorphism with experience.

The final question about which the analogy of Gödel's theorems may give us some guidance is that of decidability. The 'decision problem' for any system is whether there is a procedure which will decide the truth or falsity of *any* question which can be raised in the system; Gödel resolves the problem for arithmetic or any larger formal system with a clear 'no, there can be no such procedure'. Analogously then, we may ask, does the Bible enable any Christian to decide *any* question he may ask about what he should believe and do? If human behaviour were a closed system to which the word of God could be addressed in a finite set of rules, we might answer 'yes', but this is surely not the case. Human culture, environment, technology, understanding are all in continual flux and development, and divine revelation and instruction will need to be developed correspondingly. No static document, historically conditioned, can embrace this; we need the dynamic Spirit to interpret and apply the Word. Scripture is normative, but not exhaustive. Finally, the Christian must come back to the words of John:[46] 'In the beginning was the Word . . . the Word was God . . . the Word became flesh . . . and we saw his glory.' Jesus Christ is *the* Word of God, and all other words must be heard in the light of him.

45 Stein, J. and Taylor, H. G., *In Christ all things hold together* (Collins, 1984).
46 John 1:1, 14.

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