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On Numbers in Numbers

The apparently very large numbers of Israelites at the time of the Exodus, as recorded in the book of Numbers, have been a subject of much debate. This paper examines the recent suggestion by Prof. C.J. Humphreys that the Hebrew word 'lp can mean "troop" as well as "thousand". It is found that his approach encounters some significant problems. On the other hand, the numbers taken at face value, with 'lp consistently translated as "thousand", indicate a relatively small proportion of Israelites under the age of 20. This may have implications for explaining the low number of first-born males. Also, it suggests that the total number of Israelites was about 1.6 million.

Keywords: Exodus, Moses, census, thousand.

1. Introduction

There has been considerable discussion as to the correct interpretation of the figures recorded in the census lists of the Old Testament book of Numbers. Many scholars believe that the large numbers (e.g., 603,550 males over twenty years old in the census of Num. 1) do not accurately represent the sum of Israelites that came out of Egypt in the Exodus. The actual total is generally assumed to be much smaller.

Various solutions have been advanced to explain the large numbers in Numbers[1,2,3]. It has been proposed that the numbers are symbolic and based on gematria, that the numbers are based on astronomy and calendars, that the numbers represent the population at a much later time, that the numbers are purely fictitious, invented to serve a theological purpose, or that the Hebrew word 'lp, usually translated as "thousand", should actually be translated as "family" or "troop".

This latter view has recently been developed and promoted by Prof. C.J. Humphreys[2,3]. On the basis of his analysis he concludes that the total number of Israelites at the Exodus was only about 20,000. In this paper we shall examine Prof. Humphreys' proposal. It will be shown that his approach is plagued

1 Davies, E.W., 'A mathematical conundrum: the problem of the large numbers in Numbers I and XXVI', *Vetus Testamentum* (1995) 45, 449-69.
2 Humphreys, C.J., 'The number of people in the exodus from Egypt: decoding mathematically the very large numbers in Numbers I and XXVI', *Vetus Testamentum* (1998) 48, 196-213.
3 Humphreys, C.J., 'How many people were in the Exodus from Egypt?', *Science & Christian Belief* (2000) 12(1), 17-34.

with some serious deficiencies. We shall argue that the numbers taken at face value, with 'lp consistently translated as "thousand", indicate a much smaller proportion of Israelites under the age of 20 than is generally assumed. This may have implications for explaining the low number of first-born males. Also, it suggests that the traditional reading of Numbers implies a total number of Israelites of about 1.6 million, rather than the 2-2.5 million commonly cited.

2. Summary of Humphreys' results

In his mathematical analysis Humphreys makes the following assumptions and definitions:

1. the number of first-born male Israelites (denoted "I_f") exceeds the number of Levites (denoted "L") by 273 (Num. 3:46), or, mathematically expressed,

$$I_f - L = 273 \qquad (1)$$

2. excluding the Levites, the number of male Israelites (denoted "I") is 11 times that of the number of Levites, or

$$I = 11\ L. \qquad (2)$$

3. the male population over 20 of age (denoted "I_{20}") is half the total number of males, or

$$I = 2\ I_{20} \qquad (3)$$

4. the average number of males per family (denoted "n") is

$$n = I / I_f \qquad (4)$$

These assumptions and definitions can easily be shown to yield the relations:

$$n = 22\ I_{20} /(3003 + 2\ I_{20}) \qquad (5)$$

$$L = 273\ n /(11 - n) \qquad (6)$$

Taking 'lp to mean "troop", Humphreys interprets the first census (Num. 1) to yield a total I_{20} of 5550 males above 20 for the 11 tribes. By assumption 3, this corresponds to 11,100 males in the 11 tribes; by assumption 2 that, in turn, yields 1009 Levite males. The number of Levite males can be calculated also by using equations (5) and (6), which incorporate the further assumptions 1 and 4. Equation (5) translates the 5550 males above 20 to an average n of 8.658 males per family and, plugging this into equation (6), a total of 1009 Levite males. Remarkably, these two estimates for L are exactly the same! (Humphreys erroneously obtains an L of 1041 here, but the correct value of 1009 fits his theory even better.) Moreover, this agrees very well with the actual L of 1000, based on Humphreys' interpretation of Num. 3, again taking 'lp as "troop".

The close agreement of these results leads Humphreys to conclude that his assumptions appear justified and that his interpretation is highly self-consistent.

3. An analysis of Humphreys' theory

Although this is, at first sight, an ingenious solution and although the above results look very promising, a closer examination reveals a number of serious problems.

3.1 The implications of large family size

Humphreys justifies an average family having 8.7 sons at the time of the Exodus by noting that the Israelites had "multiplied greatly" (Exod. 1:7). However, this text refers to the state of affairs many years before the Exodus. The narrative reads:

> "But the Israelites were fruitful and multiplied greatly and became exceedingly numerous, so that the land was filled with them…But the more they were oppressed, the more they multiplied and spread; so the Egyptians came to dread the Israelites" (Exod. 1:7-12).

The text records that, years before the birth of Moses, which in turn was 80 years before the Exodus, the Israelites were already sufficiently numerous to frighten the Egyptians.

Now, counting a generous 40 years to a generation, at the time of Moses' birth the number of males over 20 would be, on Humphreys' assumptions, only 74 (i.e., $5550 / 8.7^2$). This is little larger than the number of Israelite males that initially entered Egypt with Jacob. Hardly a force that would worry the Egyptians. Indeed, at Humphreys' rate of 8.7 sons per family, Jacob's original 57 grandsons would have multiplied to the 600,000 males of the traditional reading of Num. 1 in a mere 4.3 generations. Counting 40 years to a generation, this amounts to only 172 years.

3.2 The proportion of males over 20

Moreover, such large family sizes contradict Humphreys' assumption that half of the population was aged over 20. With the above large family size, and again counting 40 years to a generation, the proportion of males over 20 would be roughly 0.34 (i.e., $8.7^{-20/40}$). Thus 5550 males over 20 corresponds to an estimated total of 16,324 (i.e., 5550 / 0.34) male Israelites and 1484 (i.e., 16324 / 11) Levites. The latter number no longer agrees well with Humphreys' interpretation of the Levite census of 1000.

Modifying Humphreys' assumption 3 to the more consistent formula $I = n^{20/40} I_{20} = n^{1/2} I_{20}$, rather than $I = 2 I_{20}$, the suitably revised equation (5) yields n = 9.3. With this new value for n, equation (6) yields 1493 Levites. On the other hand, the total number of Israelite males is now estimated to be 16,925 (i.e., $9.3^{1/2}$ x 5550), which results in 1539 (i.e., 16925 / 11) male Levites. The agreement between the two estimates for L is not quite as good as before. Neither of these concur with Humphreys' reading of 1000 Levites in Num. 3.

One could try to save Humphreys' hypothesis by taking the number of Levites to be other than I/11. In terms of Humphreys' numbers, the fractional sizes of the other tribes vary from 1/8 for Dan to 1/28 for Manasseh.

Replacing equation (2) with I = 8 L, the revised equation (3) yields a family size n of 7 and equation (6) gives L = 1911. The total number of males then becomes I = 14,684 (i.e., $7^{1/2}$ x 5550), leading to 1835 (i.e., 14684 / 8) for the second estimate of L. These estimates, although fairly consistent with each other, are far from the 1000 Humphreys obtains from the Levite census.

For I = 28 L we obtain n = 21.6 and, from equation (6), L = 921. The total number of males becomes I = 25,794 (i.e., $21.6^{1/2}$ x 5550), leading to L = 921 (i.e., 25794 / 28). These two estimates for L match exactly and agree fairly well with the 1000 from the Levite census. Nevertheless, 21 sons per family is too large to be realistic and aggravates the problem noted in the previous section.

3.3 The second census

There is a further difficulty. According to Humphreys, 40 years after the exodus, at the second census (Num. 26), there were 5730 Israelites over 20. These would almost all be under 60, since all those aged over 20 at the first census were to die (Num. 14:29), except for Joshua and Caleb. Humphreys estimates that, at the first census, there were about 5550 Israelites under 20. If the bulk of these were still alive forty years later, at the second census, then only some 200 surviving sons could have been born in the first 20 years after the Exodus. This is a tiny amount compared to the 5550 sons estimated by Humphreys to have been born in the 20 years before the Exodus. Why the sudden drastic drop?

The problem becomes much more acute if, on the basis of 8.7 sons per family, we take the more appropriate number of 10,774 (i.e., 16324 - 5550) males under 20 at the first census (see above). In that case we must conclude that most of these young men died before the second census.

One could assume that a large fraction of children died during the forty years in the wilderness. But there is little evidence for this. The adults (i.e., all those over 20) were to die for their unbelief and even that took place over 40 years. So their natural lifespan need not have been greatly shortened. However, God promised (Num. 14:31) that He would bring the little ones into the promised land.

3.4 The Levite census

Finally, we cite one last problem. According to Humphreys, the census figures for Levite men over 1 month (Num. 3:21-39) add up to 1000. However, he apparently overlooks the fact that these do not fit with the census figures for Levites between 30 and 50 (Num. 4:34-44), as shown in Table 1.

Table 1.

Number of Levites at the first census (Num. 3:21-39 and Num. 4:34-44)

	Males over 1 month		*Males 30 – 50*
Gershon	7 'lp and	500	2 'lp and 630
Kohath	8 'lp and	300	2 'lp and 750
Merari	6 'lp and	200	3 'lp and 200
Total	22 'lp (Num. 3:39)		8 'lp and 580 (Num. 4:48)

Humphreys explains the 22 'lp in the total of the first column by arguing that originally the total would have read "21 'lp (troops) and 1 'lp (thousand)". At a later date, when the original meaning was allegedly lost, a scribe is presumed to have conflated the two 'lp figures to yield 22 thousand, not realizing that 'lp had initially been used in two different senses.

However, the numbers in the first column (i.e., all males over 1 month) must clearly be larger than those in the second column (i.e., males of ages 30-50). It follows that at least one 'lp in each entry in the first column must similarly be translated as "thousand", yielding a first column total of at least 4000 Levite males over 1 month.

Moreover, Humphreys assumes that half the population is under 20. Since we can expect roughly twice as many males over 20 as between 30 and 50, we should then estimate entries in the first column to be approximately 4 times as large as the corresponding entries in the second column[4]. This results in at least 6000 Levite males over 1 month. Assuming the other tribes are as numerous as the Levites, the total number of Israelites – men, women, and children – is then at least 144,000 (i.e., 6000 x 12 x 2) at the time of the Exodus. These numbers become even larger if we take a continuous population growth equivalent to Humphreys' 8.7 sons per family. Then the ratio of those between 30 and 50 to the rest of the population is at least 7, resulting in 11,000 (i.e., 7 x 1580) Levites and 264,000 (i.e., 11000 x 12 x 2) Israelites in total.

This revised sum is much larger than Humphreys' original estimate of 20,000 Israelites. Its reconciliation with the census figures for the Israelites requires substantial ad hoc separations of the 'lp's as "troops" and "thousands". Also, it militates against the original goal of Humphreys' theory, which was to significantly reduce the apparently large number of Israelites.

4 This point has been addressed also by Heinzerling, R., 'On the interpretation of the census lists by C.J. Humphreys and G.E. Mendenhall', *Vetus Testamentum* (2000) 50, 250-251.

4. Discussion

In summary, Prof. Humphreys' proposal faces serious difficulties that seem hard to surmount. This suggests that there may be merit in reconsidering the simpler thesis that the census numbers be taken at face value, translating 'lp as "thousand" consistently whenever it is in a clearly numerical context.

Of course, this brings us back to the initial problem of explaining the large number of Israelites. It should be noted, however, that the numbers thus interpreted are, at least, internally consistent. Moreover, they accord well also with the above-cited facts of the rapid growth of the Israelites to a size sufficiently large to frighten the Egyptians (cf. Exod. 1).

What, then, about the texts Humphreys cites in support of a small number of Israelites? The verses (Exod. 23:29-30) adduced to show there were too few Israelites to occupy the promised land actually refer to a region much larger than Canaan (Exod. 23:31). And the reference (Deut. 7:7) to the Israelites being "the fewest of all peoples" in fact refers to their number when chosen by God, and could well refer to the time of Abraham.

Taking the census numbers at face value, what was the total number of Israelites? It is often assumed that there were, roughly, as many males under 20 as above 20 and as many females as males. That would lead to an estimate of about 2.4 million.

The information contained in the Levite census (see Table 1) suggests a more accurate estimate is possible. These data are very interesting, since they give both the numbers of Levite males between 30 and 50 years of age and the total number of Levite males over 1 month. From this one can make a more direct estimate of the total Israelite population at the time of the Exodus.

The fraction of Levite males between 30 and 50 is 0.39 (i.e., 8580 / 22000). This is much higher than what one might expect on the basis of continual population growth. At 8.7 sons per family one would expect at most about 0.13 of the population to be in the 30 to 50 age bracket. Even at a modest 2 sons per family this fraction would be still less than 0.25. The fact that this fraction is large for all three groups – Gershon (0.351), Kohath (0.333), and Merari (0.516) – suggests that it is no mere statistical anomaly but is reflective of the entire Israelite population.

Whether the population is stable, growing, or declining, one would normally expect at least twice as many men in the 20 to 60 age group as in the 30 to 50 range (although this is clearly not quite the case for the Merari clan). This gives about 17,160 (i.e., twice 8580) Levite men in the 20 to 60 range, leaving less than 4840 (i.e., 22000 - 17160) Levite males less than 20 or over 60, only a fraction of 0.22 of the total. If this is typical of the other tribes, then one would expect 603,550 males over 20 to be augmented by at most 170,232 (i.e., 603,550 x 4840/17160) males aged less than 20. This number is an overestimate because the 603,550 represents all males over 20, rather than just those from 20 to 60,

as in the Levite calculation. The total number of males is then estimated to be less than 774,000. Adding to this the 22,000 Levite males, we obtain a total of at most about 0.8 million Israelite males. Assuming an equal number of females yields a grand total of about 1.6 million Israelites. This number, although still very large, is significantly smaller than the 2 to 2.5 million estimate commonly given[1,2].

The small fraction of males under 20 (i.e., 0.22 minus the fraction of those over 60), being substantially less than the fraction of males between 30 and 50 (i.e, 0.39), indicates an under-representation of males under 20. It implies that, in the previous 20 years, either the birth-rate was significantly reduced or the mortality rate for male children was greatly increased.

This may be of importance regarding the perplexing problem of the number of first-born sons. A major objection to taking the census figures at face value is the difficulty in explaining the small number of 22,273 first-born sons out of a population of more than 600,000 males over 20. Presumably, the "first-born" refers to sons in each household, who would generally be under 20. Given the above 170,232 males under 20, this works out to 7.6 sons per family, which is not impossible. But this raises the question as to why there were only 22,273 fathers (most, presumably, in the 20-40 age group) among the 600,000 males above 20. Could it be that, due to the heavy oppression in Egypt, few married in the decades just before the Exodus? Or, perhaps, that many of the first-born infant sons had died (cf. Exod. 1:22), leaving only a remnant? Both of these explanations are consistent with the small fraction of males under 20.

Harrison[5] considers the possibility that the first-born applies only to males born in the first 12 months after the Exodus. In that case 22,273 first-born in one year seems rather high. One could postulate, perhaps, that many couples, who had postponed marriage during the years of oppression, married around the time of the Exodus in anticipation of blessings in the promised land. This has the merit of explaining the low redemption price of 5 shekels (Num. 3:47), which is elsewhere specified (Lev. 27:5-6) to be 5 shekels for males over one month and less than 5 years, increasing to 20 shekels for males between 5 and 20 years old.

All of these scenarios are admittedly speculative. Nevertheless, they suggest that the number of first-born can be plausibly explained within a context that takes the large numbers as historically accurate.

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5 Harrison, R.K *Numbers*, Grand Rapids: Baker (1992).

Response by Colin J. Humphreys

I am grateful to Professor Byl for his detailed mathematical analysis of my paper in S & CB and for the many good points that he makes. However, I do not believe his analysis is appropriate to the situation at the time of the Exodus and I would like to focus my response on the main point that he makes in his section 3.1: the implications of large family size. First, we need to go back to the biblical account and see exactly what it says.

Exodus chapter 1 records that the Egyptians were greatly concerned and threatened by the rapidly increasing population of the Israelites; hence the King of Egypt ordered the Hebrew midwives to kill every new born boy, but to let the girls live. The Hebrew midwives cunningly avoided doing this. "Then Pharaoh gave this order to all his people: 'every boy that is born you must throw into the Nile, but let every girl live'." Since Pharaoh ordered his own people to kill all the Hebrew baby boys, the strong implication of this verse is that this really happened, since Egyptians would not disobey their own Pharaoh. The Egyptians therefore performed a terrible slaughter of all the male Israelite babies. However, the baby Moses amazingly escaped death as described in Exodus chapter two.

We do not know for how long the Egyptian slaughter of all male Hebrew babies continued, but it is reasonable to assume that it lasted for a sufficient length of time to reduce the numbers until they were no longer seen by the Egyptians to be a threat, after which the killing would have stopped, because the Egyptians did not want to eliminate totally their valuable slave labour force. The Israelites would then have had a severe population imbalance, with many more females than males. Almost certainly the Hebrew males would have practised polygamy, for the benefit of the whole of the Israelite community at this time of population imbalance (the evidence from the Old Testament is that in ancient Israel polygamy was widely practised and, for example, Abraham, Isaac and Jacob all had more than one wife).

The above is consistent with my analysis that at the time of the Exodus, two generations after the birth of Moses, there was an average of 8.7 sons per family, i.e. about 17 children per family. This could only have been achievable if each man had several wives. However, the mistake that I believe John Byl makes in interpreting my figures is to assume that the Israelites had 8.7 sons per family over the whole period of time, two generations, from the birth of Moses to the Exodus. If my interpretation of the book of Exodus given above is correct, then there were clearly not 8.7 sons per family at the time of the birth of Moses because Pharaoh had ordered all sons to be killed at birth. Hence, when John Byl writes "at the time of Moses' birth the number of males over 20 would be, on Humphreys' assumptions, roughly $5550/8.7^2$ or 73 … hardly a force that would worry the Egyptians", this is wrong because it takes no account of the effects of the slaughter of the male babies that was occurring at the time of the birth of Moses.

I suggest that the population situation was broadly as follows. Before Moses was born the Egyptians were extremely worried by the large number of male Israelites. We do not know how large this was, but for the sake of argument, let us say there were 20,000, which would be a potential fighting force that would worry the Egyptians. The Egyptians then started to kill all, or most of, the Israelite boys that were born. We do not know for how many years this went on, but, for the sake of argument, let us say the Israelite male population was reduced to 1000 men. The Egyptians then stopped the slaughter and the population increased to 11,000 men at the time of the Exodus (about 5550 of these over the age of 20) with 8.7 sons per family. I suggest that this picture is qualitatively consistent with the Exodus account. The mathematical analysis John Byl has performed is not appropriate in these circumstances. Similarly, the rest of John Byl's analysis of my numbers assumes a constant rate of 8.7 sons per family over time which is not appropriate. My analysis gives 8.7 sons per family specifically at the time of the Exodus and it does not give the number of sons per family at an earlier time. I would also like to point out that my analysis is an approximate one. The figure 8.7 comes from the mathematics, but I certainly would not claim this degree of precision!

John Byl rejects my analysis and suggests that the total number of Israelites at the Exodus was about, or less than, 1.75 million. I pointed out in my previous paper the very substantial difficulties in accepting such a large number. A further reason relates to the crossing of the Red Sea, which the book of Exodus records happened in less than one night. 1.75 million people, ten abreast and 1 metre apart, would form a column of people 175 kilometres long. It is hard to believe that so many people could cross the Red Sea on foot in one night. In addition, if there were 1.75 million Israelites at the Exodus, then the Egyptians must have been singularly unsuccessful in their policy of killing the infant Hebrew males to reduce the Israelite population. The book of Exodus gives no indication that this Egyptian policy was unsuccessful, indeed it portrays Moses as being specially preserved from the infant culling and carnage. I therefore suggest that my figure of 20,000 men, women and children involved in the Exodus is more consistent with the biblical account than 1.75 million.

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